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MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 10, July 30, 2018

1. A sports club has a total of 163 members. The club offers a choice of basketball, cricket and soccer (or a combination of these three). Each member selects one or more activity to do. Of the club members, 73 play cricket, 100 play basketball, 60 play at least two different sports and 10 play all three.

   (a) How many members play soccer?

   (b) There are 25 members that play both basketball and cricket, how many members play soccer *only*?

2. (a) Show that if a whole number is divisible by 4, then so is the number formed by its last two digits.

   (b) Show that if a whole number is divisible by 9, then so is the sum of its digits.

3. Show that, for any whole number $n > 1$,

$$\frac{n-1}{n} - \frac{n-2}{n-1} = \frac{1}{n(n-1)}.$$

   Hence calculate

$$1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{10100}.$$

4. A continued fraction has the form

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}},$$

   where a_0 is a non-negative integer and $a_1, a_2, a_3, \ldots$ are positive integers. It is generally easier to write a continued fraction in the form $[a_0; a_1, a_2, a_3, \ldots]$. A continued fraction may either terminate or be an infinite expression.

Question 4 continues over the page.

Find the continued fraction expression for the following numbers. What do you notice?

(a) $\frac{355}{113}$

(b) $\frac{113}{355}$

(c) $\sqrt{2}$

(d) $\frac{1}{\sqrt{2}}$

Hint: a_0 *is the integer part of the number.*

5. (a) Let ABC be an isosceles triangle with BC the base. Let D

Senior Questions

1. A polynomial (of degree n) is a function that can be written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

where $a_n \neq 0$. That is, a polynomial is the finite sum of monomial terms of the form $a_k x^k$ in which the variable x is raised to a non-negative integer power. The largest value of k for which $a_k \neq 0$ is called the degree of the polynomial.

Let p and q be non-zero polynomials.[2]

(a) Show that $\deg(p + q) \leq \max(\deg p, \deg q)$. When would strict inequality hold?

(b) What is the relationship between $\deg(p \circ q)$ and $\deg(q \circ p)$? Note: $p \circ q(x) = p(q(x))$.

(c) Show that, for $x > 0$, the function $f(x) = \log x$ is not a polynomial.
Hint: consider $\log(x^2)$.

[2] This question is adapted from EJ Barbeau, *Polynomials*, Springer, 1989.