

with it (1, 2, 3 and 4), so  $\phi(5) = 4$ . In contrast, there are only two integers less than 6 that are coprime with it (1 and 5), so  $\phi(6) = 2$ .

(a) Find  $\phi(12)$  and  $\phi(30)$ .

(b) Suppose that  $p$  is prime. Find  $\phi(p)$ ,  $\phi(p^2)$  and  $\phi(p^3)$ .

(c) If  $p$  and  $q$  are two distinct primes, find  $\phi(pq)$ .

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is a triangle with  $E$  and  $F$  being the  
Prove that  $EF$  is parallel to  $BC$  and half the

### Senior Questions

- (a) Show that  $n^4 - 6n^3 + 18n^2 + 6n + 1 = (n^2 - 3n + 1)^2 + 25n^2$ ,

(b) Hence find all integers  $n$  such that  $n^4 - 6n^3 + 18n^2 + 6n + 1$  is prime.
- How many real roots does the equation  $x = 3(1 - \sin x)$  have? Use Newton's method to find an approximate value of the smallest one and hence find the largest one.
- Let  $ABC$  be a triangle. The median of a triangle is the line segment that connects the midpoint of one side to the opposite vertex. Prove that the medians of  $ABC$  intersect at a single point, called the centroid, and that the centroid divides the median in the ratio  $1 : 2$ , with the centroid lying twice as far from the vertex as from the foot of the median.