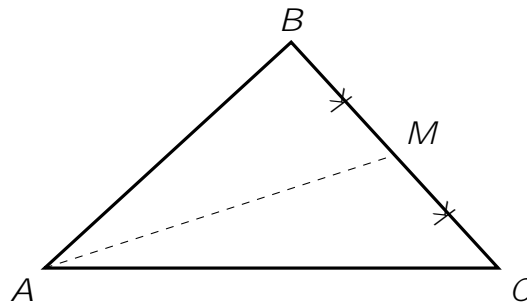


**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 9, July 2, 2018**

1. The angles in a triangle are in the ratio 2 : 3 : 4. Find, in degrees, the size of the largest angle.
2. How many digits does the number  $125^{100}$  have?
3. Let  $ABC$  be a triangle with  $AM$  one of its medians.



Prove that  $AM$  is smaller than the semi-perimeter of  $\triangle ABC$ . That is, show that  $AM < \frac{1}{2}(AB + BC + AC)$ <sup>1</sup>.

4. Let

$$= \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

Evaluate .

5. (a) Find the greatest common divisor of  $2^{50} + 1$  and  $2^{20} + 1$ .
- (b) Explain why the greatest common divisor of  $2^m + 1$  and  $2^n + 1$  is at least three if  $m$  and  $n$  are both odd.

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<sup>1</sup>This question is adapted from A. P. Kiselev, *Geometry: Planimetry*, tr. A Givental, 2006

## Senior Questions

1. **The Miquel Point.** Let  $ABC$  be a triangle. Let  $D$ ,  $E$  and  $F$  be points on the sides of the triangle. Show that circles through  $ADE$ ,  $BDF$  and  $CEF$  intersect at a common point  $G$  as shown. <sup>2</sup>

