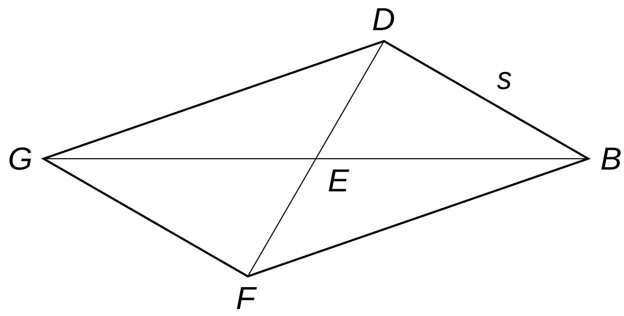
[redacted]

MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 15, September 10, 2018

1. If Cog-1 rotates clockwise, Cog-2 must rotate counter clockwise, and so Cog-3 must

(v) Extend DE and BE.

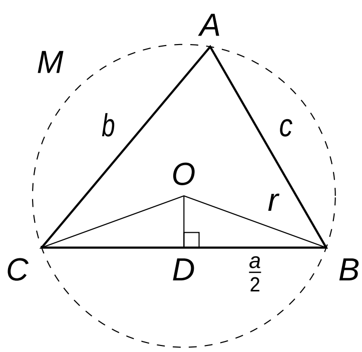
(vi) Using the compasses, nd point F on DE such that $EF = DE$, and point G on BE such that $EG = BE$.



Then DF and BG bisect each other and hence $DBFG$ is a parallelogram. Moreover, $DF + BG = d$, the angle between DF and GB is , and the length of the side DB is s. Thus $DBFG$ has the required properties.

4. If a number is written in its prime factorisation $n = p_1^{m_1} p_2^{m_2} ::: p_k^{m_k}$, then for it to be powerful each of the m_i 2 and for it to be a perfect power all $m_i = c$, a constant. Thus for n to be powerful but not a perfect power all the m_i must be greater than 2, but not all the same. The smallest then, would be 2^3 3^2 = 72.

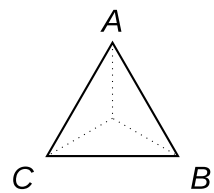
5. Let O be the centre of M, and let D be the midpoint of BC.



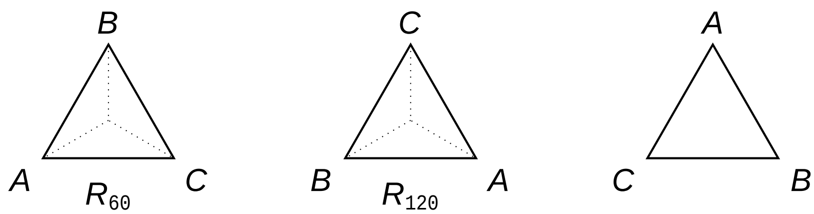
(a-not 81q288 -46.025 cmQ Q q q 1 0 0 1 35.66 22.322 c806051s4.t326.02243 [(3 -22.322 0 Td81 343.6

Senior Questions

1. (a) Consider the following triangle, which has its vertices labelled *A*, *B*, *C* in a clockwise fashion from the top. We will consider this as the initial position of the triangle.



Then there are three rotations (measured in the counter-clockwise direction), which I will designate R_{60}, R_{120} and R_{360}.



Interestingly, there is a subset of the operations that do commute with each other. Can you see which ones they are?

(c) Obviously, this is R_{360}, the \do nothing" operation. (I could also have called it R_0.)

(d) Clearly, R_{360} is it's own inverse, as are the three ipping operations | F_{90}, F_{210} and F_{330}. The two other rotations, R_{60} and R_{120}, are inverses of each other.