MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 16, September 17, 2018

1. It requires two people to shake hands. According to the guests' claims, we see that there have been exactly 5 11 = 55 instances of people taking part in one half of a handshake. As this is not an even number, it cannot be twice the total number of handshakes. Thus someone is lying.

2. In the 3 3

4. This is basically a proof by exhaustion of cases. A two-digit narcissistic number with digits *ab* must satisfy

$$a^2 + b^2 = 10a + b;$$

or

$$b^2 \quad b + (a^2 \quad 10a) = 0:$$

We can consider this as a quadratic in *b*, with discriminant

$$= 1 \quad 4(a^2 \quad 10a) = 101 \quad 4(a \quad 5)^2:$$

If *a* is an integer between 1 and 9, we obtain the following values for   :

a	
1	37
2	65
3	85
4	97
5	101
6	97
7	85
8	65
9	37

As none of these values is a perfect square, *b* is an irrational number in all cases. So there are no 2-digit narcissistic numbers.

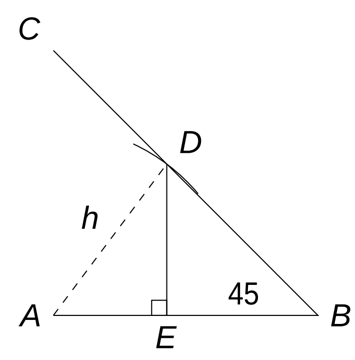
h 5. (a) Suppose that we are given the length of the hypotenuse *h* and the sum of the two short sides, *s*.

(i) Construct a line *AB* equal to *s*.

(ii) Construct a ray, *BC*, at an angle of 45 to *AB* at *B*.

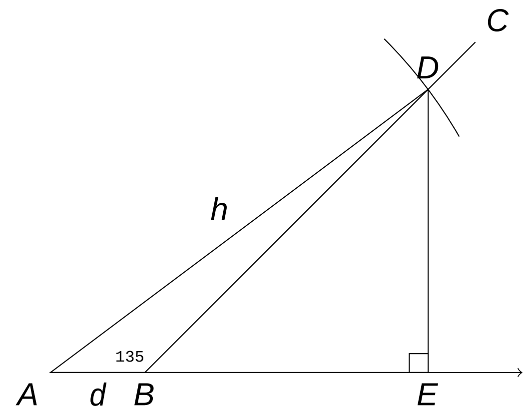
(iii) Using the compasses, draw an arc with radius *h* centered at *A*. Let *D* be the point where this arc intersects *BC*. (NB: two possible positions for *D*.)

(iv) Drop a perpendicular from *D* to *AB*. Let the foot of this perpendiular be *E*. Then 4*ADE* is the desired triangle.



E4*ADE* is rig27(ypt-gle.)d3227abosituse

(b) Suppose that we are given the length of the hypotenuse h and the difference of the two short sides, d.



(i) Construct a line segment AB with length d.

(ii) Construct a ray, BC, at an angle of 135° to AB at B.

(iii) Using the compasses, find a point D on BC that is a distance of h from A.

(iv) Extend AB and drop a perpendicular from D

Using the remainder theorem, we can easily con rm that $x = \frac{1}{2}$ is a solution to this polynomial. Then using polynomial long division, we can show that

$$8x^3$$