[redacted]

MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 4, June 4, 2018

1. Since x is an integer, x^2 is the product of even powers of 2 and 3, and hence y^3 is also a product of even powers of 2 and 3. Then y^3 can be 1, 2^6, 2^{12}, 3^6, 3^{12}, $2^6\ \ 3^6$, $2^6\ \ 3^{12}$, $2^{12}\ \ 3^6$ or $2^{12}\ \ 3^{12}$. For each of these y values, there is one value of x. Hence there are 3^{12}nine solutions altogether.

4. The sum of the digits $1, 2, 3, \dots, 9$ is 45 [(1 + 9) + (2 + 8) + ... + 5]. Also r[redacted]
if we have a sum like $\sum_{k=0}^{n}(a+k) = a(n+1) + \sum_{k=0}^{n} k$, then the required su[redacted]

$$\sum_{a=0}^{9}\sum_{b=0}^{9}\sum_{c=0}^{9}\sum_{d=0}^{9}(a+b+c+d) = \sum_{a=0}^{9}\sum_{b=0}^{9}\sum_{c=0}^{9} 10(a+b+c) \sum_{d=0}^{9} d$$ [redacted]

$$= \sum_{a=0}^{9}\sum_{b=0}^{9}\sum_{c=0}^{9}(45 + 10a + 10b + 10c)$$ [redacted]

$$= \sum_{a=0}^{9}\sum_{b=0}^{9} 10(45 + 10a + 10b) + 10$$ [redacted]

$$= \sum_{a=0}^{9}\sum_{b=0}^{9}(450 + 100a + 100b + 450)$$ [redacted]

$$= \sum_{a=0}^{9} 10(900 + 100a) + 100 \sum_{b=0}^{9} b$$ [redacted]

$$= \sum_{a=0}^{9}(9000 + 1000a + 4500)$$ [redacted]

$$= 10 \quad 13\,500 + 1000 \quad 45$$

$$= 180\,000$$

Senior Questions

1. Firstly, we complete the square in a slightly unusual way.

x^2 $\sum_{c=0}^{9}$ 0920 7.9701 Tf 42.65 14.94652 6 34.5ci667 0 Td [(=0)]TJ/F19 11.9552 Tf

10(a=0

where x and y are integers. Thus

$$y^2 + 2(x \quad 10)y = x \quad 6$$
$$y^2 + 2xy \quad 20y = x \quad 6$$
$$y^2 \quad 20y + 6 = x(1 \quad 2y)$$

So

$$x = y^2 \quad 20y + 6$$