n v rs ty o w out a s c oo Mat at cs Co p t t on [1]

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rob

A second-cousin prime n-tuple is defined as a set of n prime numbers $\{p\ p+6\ \ldots\ p+6(n-1)\}$ with common difference six. Each number in the set is a prime and consecutive members of the set differ by six. For example 2011 is a member of a second-cousin prime 2-tuple.

Show that there is one and only one second-cousin prime 5-tuple and there are no second-cousin prime 6-tuples.

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Clearly if p is not equal to five and is a member of a second-cousin prime n-tuple then the last digit of p must be one of one, three, seven or nine. Suppose it ends in one, then the next member of the second-cousin prime n

2. Select thirteen coins at random and move (slide) them into a separate group. There will then be $n \leq 13$ coins heads up and $13 - n$ coins heads down in the separated group and there will be $13 - n$ coins heads up in the remaining group. Now turn all coins over in the separated group to have $13 - n$ coins heads up.

rob

Six hundred and sixty-six students sit for a prestigious mathematics contest. It is known that all of the students who sit the exam attend an all girls school and/or play sport on the weekend, and/or play a musical instrument. One hundred and eleven of the students attend an all girls school and two hundred and twenty-two attend an all boys school. Four hundred and forty-four of the students play musical instruments and five hundred and fifty-five of the students play sport on the weekend. Seventy-seven of the students attend an all girls school and play sport on the weekend. Eighty-eight of the students attend an all girls school and play a musical instrument. Three hundred and thirty-three of the students play a musical instrument and play sport on the weekend. Of the students who attend an all boys school thirty-three of them do not both play sport on the weekend and play a musical instrument. How many of the students attend a co-ed school, play sport on the weekend and play a musical instrument?

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Let denote the total number of students who sat the competition, (G) the number who attended an all girls school, (B) the number who attend an all boys school, (C) the number who attend a co-ed school, ( ) the number who play a musical instrument, and ( ) the number who play sport on the weekends. Let ($\cap$) denote the number who play sport on the weekends and play a musical instrument etc. The following relations hold

$$
\begin{aligned}
 &= (G) + ( ) + ( ) \\
 &\quad - (G \cap) - (G \cap) - (\cap) \\
 &\quad + (G \cap \cap) \\
(B) &= (B \cap \not\cap) + (B \cap \not\cap) + (B \cap \cap) \\
(\cap) &= (G \cap \cap) + (B \cap \cap) + (\quad N
\end{aligned}
$$

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$$\frac{a}{!} \quad \sqrt{}$$

$$\Rightarrow \frac{a + !}{!} \quad \sqrt{}$$

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A standard domino is a rectangular tile with a line dividing the rectangular face into two equal-sized squares. Each square is decorated with a number of pips (including zero pips – a blank end). A Double set of dominos is composed of one each of all possibilities with the number of pips on one square end less than or equal to the number of pips on the other square end and the maximum number of pips on any square end equal to . The most common set of dominos is called the Double Six set.

1. It is well known that there are 28 dominos in a Double Six set. Show that the number of dominos in a Double 48 set is a perfect square.

2. Prove that it is not possible to tile any square region without gaps or overlaps using a complete Double set of dominos.

3. A tri-omino is an equilateral triangular tile with lines dividing the triangular face into four equilateral triangles decorated with blanks or pips. There are $\frac{1}{6}(\quad + 3)(\quad + 2)(\quad + 1)$ tri-ominos in a Triple set. Is it possible to tile a triangular region without gaps or overlaps using a complete Triple set of tri-ominos if 1?

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1. The number of dominos in a Double Six set is

$$(6 + 1) + (5 + 1) + (4 + 1) + \quad 1 = 28$$

The number of dominos in a Double set is

$$\sum_{k=0}^{N}(\quad + 1) \;=\; \quad + 1 + \frac{1}{2} \quad (\quad + 1)$$
$$= \frac{1}{2}(\quad + 2)(\quad + 1)$$

Note that if $= 48$ then

$$\frac{1}{2}(\quad + 2)(\quad + 1) = \frac{1}{2}(50)(\quad \text{NO NO861NES D+ 1718()433(0 11943128784]TJ R12 1}$$

2. Assuming the trend continues after the election what is the maximum vote, to the nearest percent, that the losing party could attain in the next four year term of government?

o ut on

Let $A(n)$ denote the proportion of the vote for the ALP at time n and let $B(n)$ denote the proportion of the vote for the Coalition at time n with one month corresponding to the unit of time.

The vote for the ALP in month $n+1$ will be

$$A(n+1) = \begin{cases} A(n) - \frac{30}{100}A(n) + \frac{20}{100}B(n) & A(n) \quad B(n) \\ A(n) - \frac{20}{100}A(n) + \frac{30}{100}B(n) & A(n) \quad B(n) \end{cases}$$

The vote for the Coalition in month $n+1$ will be

$$B(n+1) = \begin{cases} B(n) - \frac{30}{100}B(n) + \frac{20}{100}A(n) & A(n) \quad B(n) \\ B(n) - \frac{20}{100}B(n) + \frac{30}{100}A(n) & B(n) \quad A(n) \end{cases}$$

At all times we have $A(n) + B(n) = 1$ so that it is sufficient to consider the dynamics of the vote for one party, the ALP say, in which case the governing equation simplifies to

$$A(n+1) = \begin{cases} \frac{20}{100} + \frac{50}{100}A(n) & A(n) \quad \frac{50}{100} \\ \frac{30}{100} + \frac{50}{100}A(n) & A(n) \quad \frac{50}{100} \end{cases}$$

1. It is a simple matter to start with $A(0) = 0.55$ and then calculate $A(1) = 0.475$ $A(2) = 0.5375$ $A(3) = 0.46875$ $A(4) = 0.534375$ $A(5) = 0.4671875$ so that the ALP would have less than half the vote at the time of the election.

2. Note that the proportion of the vote to the ALP quickly becomes cyclic so that if $A(\ )\quad 0.5$ then $A(\ + 1)\quad 0.5$ and vice versa. Thus we can write

$$A(\ +2) = \frac{20}{100} + \frac{50}{100}A(\ +1) = \frac{20}{100} + \frac{50}{100}\left(\frac{30}{100} + \frac{50}{100}A(\ )\right).$$

   After a long time we expect $A(\ +2) = A(\ ) = X$ with solution $X = \frac{7}{15}$. Thus after long times we anticipate the vote for any one party will alternate from one month to the next between $X = \frac{7}{15}$ and $1 - X = \frac{8}{15}$. The maximum vote for the losing party if the trend continues is thus $\frac{8}{15}$ or 53.3% of the vote.

rob

Two friends pass time playing a simple game with standard dice; small cubes with the faces displaying pips that number from one to six.

a seven is = $\frac{5}{6}$. Thus similar to above the probability that the first player to roll a pair is the first player to roll a seven is $\frac{6}{11}$ and the probability that the second player to roll a pair is the first to roll a seven is $\frac{5}{11}$.

The probability that the person who rolled the single die first will win the game is thus the probability that they rolled the first six and then (being the first to roll a pair) they rolled the first seven plus the probability that they did not roll the first six and then (being the second to roll a pair) they rolled the first seven:

$$= \, 6$$

Hence

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

But this is the standard geometric series

$$= \sum_{k=0}^{\infty} \Big(1$$

Thus

$$\frac{CF}{AF} = \frac{\text{Area}(\triangle CGF)}{\text{Area}(\triangle AGF)} = \frac{\text{Area}(\triangle CBF)}{\text{Area}(\triangle ABF)}$$

But

$$\frac{\text{Area}(\triangle CBF)}{\text{Area}(\triangle ABF)} = \frac{\text{Area}(\triangle CGF) + \text{Area}(\triangle BGC)}{\text{Area}(\triangle AGF) + \text{Area}(\triangle BGA)}$$

and hence

$$\frac{\text{Area}(\triangle CGF)}{\text{Area}(\triangle AGF)} = \frac{\text{Area}(\triangle CGF) + \text{Area}(\triangle BGC)}{\text{Area}(\triangle AGF) + \text{Area}(\triangle BGA)}.$$

The result

$$\frac{\text{Area}(\triangle CGF)}{\text{Area}(\triangle AGF)} = \quad \text{) CEG} \quad =$$

The right-hand side of the above inequality then reads

$$
\begin{aligned}
\prod_{j=2}^{2011} \frac{\;^{3} - 1}{\;^{3} + 1} &= \prod_{j=1}^{2011} \frac{(\;\; - 1)\;\;(\;\;)}{(\;\; + 1)\;\;(\;\; - 1)} \\
&= \frac{2010!}{2012!} \times \frac{2\;\;(2011)}{(1)} \\
&= \frac{2}{3} \times \frac{2011^2 + 2011 + 1}{2011 \times 2012} \\
&= \frac{2}{3} \times \frac{2011 \times 2012 + 1}{2011 \times 2012} \\
&= \frac{2}{3} \times \left(1 + \frac{1}{2011 \times 2012}\right) \\
&< \frac{2}{3} \times \left(1 + \frac{1}{2010}\right) \\
&= \frac{2}{3} \times \frac{2011}{2010}
\end{aligned}
$$