

**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 10, August 6, 2019**

1. Alice and Carla are playing a dice game. Here's how it works:

Each person rolls a die, and the highest number rolled of the two is recorded.

If the highest number rolled is a 1;2;3 or 4, Alice wins.

If the highest number rolled is a 5 or a 6, Carla wins.

On average, who is more likely to win: Alice, Carla, or are the probabilities equal?

2. How many 3 digit positive integers are the sum of exactly 9 distinct powers of 2?
3. Given that  $a + b = 1$  and  $a^2 + b^2 = 2$ , what is the value of  $a^7 + b^7$ ?
4. Given that  $x$  and  $y$  are distinct, non-negative real numbers such that

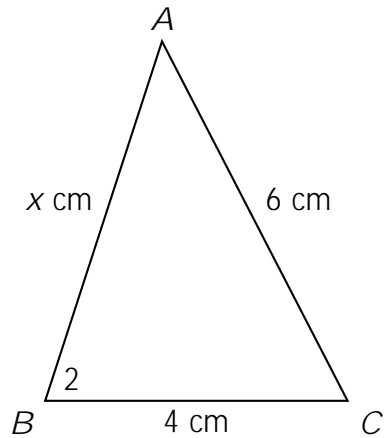
$$x + \sqrt{y} = y + \sqrt{x};$$

determine the maximum value of  $x + y$ .

5. Let  $\triangle ABC$

### Senior Questions

1. Find the remainder when  $x^{2019}$  is divided by  $x^2 - 1$ .
2. In  $\triangle ABC$ ,  $AC = 6$  cm,  $BC = 4$  cm,  $\angle A = 90^\circ$  and  $\angle B = 2^\circ$ , as shown below.



Determine the value of  $x$ .

3. Find all solutions of  $2^x + 3^x + 6^x = x^2$ .