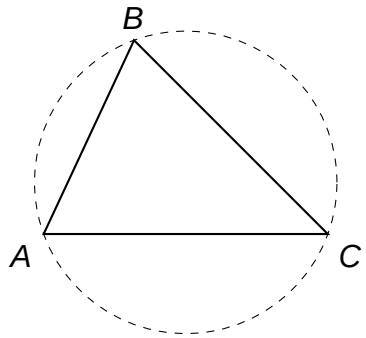
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 6, June 10, 2019[1]

1. Some simple construction problems using straight-edge and compass techniques:

   (a) Given an interval AB, describe how to construct an equilateral triangle with AB as a base.

   (b) Given an triangle ABC, describe how to construct its circumcircle. (The circumcircle is the unique circle which passes through the three vertices of the triangle.)



2. Find the number of ordered pairs $(x; y)$ of non-negative integers such that $x + y \quad 100$.

3. Let p be your favourite prime number greater than 100, and $a; b$ positive integers such that $p^2 + a^2 = b^2$. Find $\frac{a+b}{p}$.

4. At a party of 21 people each person knows at most four others. Prove that there are ve in the party who mutually do not know each other.

5. Let $f(x)$ be a polynomial with integer coe cients. Suppose $a_1; a_2; a_3; a_4; a_5$ are distinct integers such that $f(a_1) = f(a_2) = f(a_3) = f(a_4) = f(a_5) = 2015$. Find the number of integral solutions for the equation $f(x) = 2016$.

6. M is the midpoint of the side CA of triangle ABC. P is some point on the side BC. AP and BM intersect at the point O. If $BO = BP$, determine $\frac{jOMj}{jPCj}$.

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*

Senior Questions

1. Suppose that ABC is a triangle in which all internal angles are less than 120. The Fermat-Torricelli point of $4ABC$, shown as T in the diagram below, is the point inside the triangle such that $\angle ATB = \angle ATC = \angle BTC = 120$.