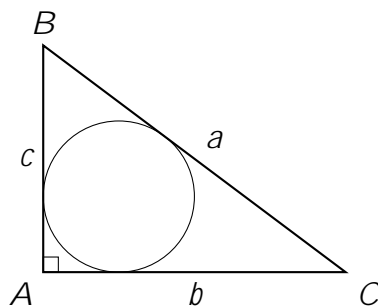


**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 7, June 17, 2019<sup>1</sup>**

1. How many pairs of integers,  $x$  and  $y$ , are there such that  $x^2 - y^2 = 2019$ ?
2. Let  $\triangle ABC$  be a triangle with sides of length  $a$ ,  $b$  and  $c$  and  $\angle A = 90^\circ$ . The *incircle* of a triangle is a circle inside the triangle that is tangent to each side. (See diagram below.) Find the radius of the incircle of  $\triangle ABC$  in terms of  $a$ ,  $b$  and  $c$ .



3. Let  $N$  be a number of the form  $N = \frac{333 \dots 333}{61}$

## Senior Questions

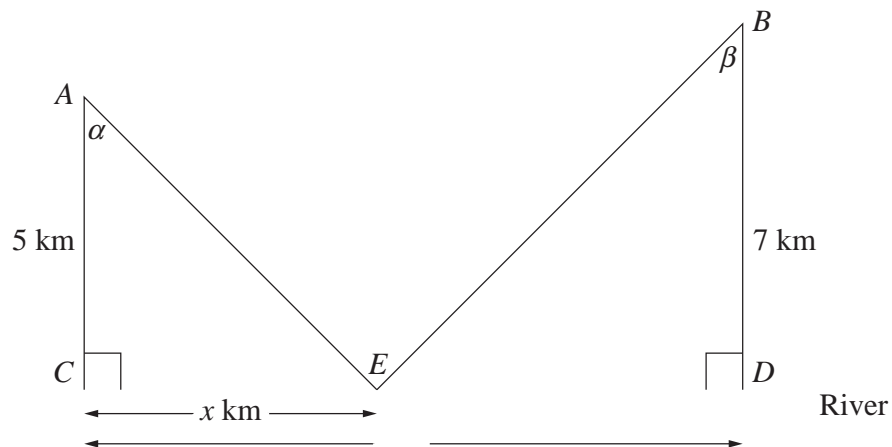
1. Consider the quadratic equation

$$f(x) = x^2 - 2(c+1)x + c - 3;$$

where  $c$  is some real number. Let  $\alpha > 0$ , and suppose  $\alpha + \frac{1}{\alpha}$  and  $2 - \frac{1}{\alpha}$  are the roots of  $f(x)$ . Find all possible values for  $c$ .

2. Given that  $\frac{a}{b} = \frac{c}{d}$  and  $c$  are positive integers, find the conditions for which the equation  $\frac{a}{b} = \frac{c}{d}$  has a solution.
3. The following is Question 16(a) from the 2017 HSC Mathematics paper. (You don't have to work through the question, but you should read it.)

To get some exercise, John cycles from home directly to point  $E$  on the river,  $x$  km from  $C$ , before cycling directly to school at  $B$ , as shown in the diagram.



$$\sqrt{\quad} \quad \sqrt{\quad}$$

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In this question, calculus is used to find the shortest length of the path  $A \rightarrow E \rightarrow B$ . There is a much simpler method for solving this problem that does not require the use of calculus. See if you can figure out what it is.