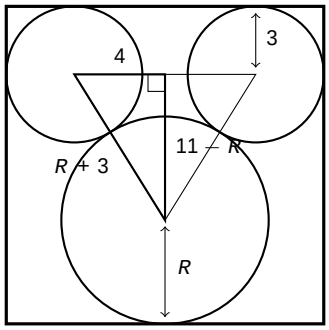
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 11 Solutions, August 20, 2019

1. Firstly note that

$$\frac{n^2 + 11n + 2}{n + 5} = \frac{n^2 + 11n + 30 \quad 28}{n + 5}$$
$$= \frac{(n + 5)(n + 6)}{n + 5} \quad \frac{28}{n + 5}$$
$$= (n + 6) \quad \frac{28}{n + 5}$$

This means that the LHS is an integer if $(n + 5)$ is a factor of 28. The positive factors of 28 are $1; 2; 4; 7; 14; 28$, so the positive solutions for n are $2; 9; 23$.

2. Let R be the radius of the big circle. Draw a triangle that connects the centre of each circles, then bisect the this triangle into two right-angled triangles; as shown in the diagram.

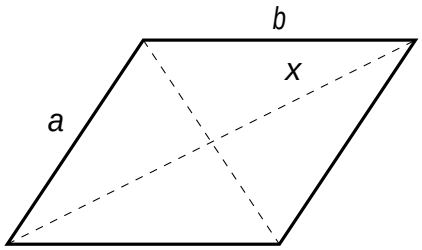


From the diagram, we can see that the hypotenuse of the right-angled triangle has length $R + 3$, and short sides of length 4 and $14 \quad R \quad 3 = 11 \quad R$. Now by Pythagoras, we have

1; 2;

Senior Questions

1. (a) Let one of the internal angles of the parallelogram be . Then the other internal angle is 180° .



2. The example shows that -4 is in T. We have further that -1 is in T, because $-1 = (5 - 4) = (0 + 1)$. Also 3 is in T, because $3 = (4 - 1) = (0 + 3)$. Continuing in this way, we can eventually obtain $f-5, -4, \ldots, 4, 5\} \subseteq T$; that is the integers from -5 to 5 are all elements of the set T.