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MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 15 Solutions, September 17, 2019

1. Let $QS = x$ and $SP = y$. We want to find the value of

Since y is an integer, this implies that $x - 4$ is a factor of 80. As both x and y are positive, this means that the possible solutions are $(5, 120)$, $(6, 84)$, $(8, 72)$, $(12, 78)$, $(14, 84)$, $(24, 120)$, $(44, 198)$, and $(84, 357)$.

3. If the number is made from $a \neq 1$, then $aaa\ldots$ is divisible by a and thus not prime.

   Suppose the number has a non-prime number of digits, then we can factor the number of digit this number has into $q \times p$ where q and p are integers. Hence, we we can "split" the number up into q blocks of p-length digits; i.e

$$\underbrace{111\ldots1}_{p \times q \text{ lots of 1's}} = \underbrace{\underbrace{111\ldots1}_{p \text{ lots of 1's}}\ \underbrace{111\ldots1}_{p \text{ lots of 1's}}\ \ldots\ \underbrace{111\ldots1}_{p \text{ lots of 1's}}}_{q \text{ lots of blocks}}.$$

   The RHS of the number above is divisible by $\underbrace{111\ldots1}_{p \text{ lots of 1's}}$.

4.

Senior Questions

1.

$$T_n \quad \frac{1}{T_1} + \frac{1}{T_2} + \cdots + \frac{1}{T_n} \quad = n^2$$

or

$$\frac{1}{T_1} + \frac{1}{T_2} + \cdots + \frac{1}{T_n} = \frac{2n}{n+1}.$$

This can be proven using induction. The inductive step depends on

$$\frac{2n}{n+1} + \frac{1}{(n+1)(n+2)=2} = \frac{2(n+1)}{(n+1)+1}.$$

2. Firstly, let's nd the equation of the chord AB. Since this line passes through $A(a; a^2)$ and $B(b; b^2)$, the gradient is given by

$$m_{AB} = \frac{a^2 \quad b^2}{a \quad b} = a + b.$$

Using the point gradient form of a line,

$$\begin{aligned} y \quad y_0 &= m(x \quad x_0) \\ y \quad a^2 &= (a+b)(x \quad a) \\ y &= (a+b)x \quad ab. \end{aligned}$$

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