



# 7ci fgY<sup>\*</sup>Cj Yfj ]Yk

# GHJZZ:7cbHJWh:8YHJ]`g

## 7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] <b>h</b> m	@cWUh]cb	D\cbY
A UHNYK '6fcX]Y	h\Yg]g"V]ca YXYb[ 4 i bgk "YXi " <u>Ui</u>	VmUddc]bha Ybh	GUaiY`gʻ)%)	
H]Ubfic; ic	ht[ic4ibgk "YXi"Ui	VmUddc]bha Ybh	GUaiY`g')%)	

## GWXcc`7cbHJWhi±bZcfa Uhjcb

Gh XYbhGYfj ]WYg WUb VY WcbhUWYX j ]U i bgk "hc#k YVZcfa g"

## 7ci fgY 8YHJ] g

#### I b]hg~cZ7fYX]h~(~

#### Gi a a UfmcZh\Y 7ci fgY

H\Y'h\Yg]g'dfcj ]XYg'Ub'cddcfhi b]miZcf'mci 'hc'Vf]b[ 'hc[Yh\Yf'Yb[]bYYf]b[ 'df]bW]d'Yg'YUfbYX'cj Yf mci f'dfYj ]ci g'mYUfg'cZghi XmUbX'Udd'mh\YgY'df]bW]d'Yg'hc']bbcj Uhji Y'mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y'cda YbhcZU'gdYW]ZJWXYg][ bž'dfcWYgg'UbX#cf'h\Y']bj Ygh][ Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc YWg'UfY Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU`ck 'fcca 'Zcf'mci f'WfYUhji ]mž'UbX'h\Y'UWei ]g]hjcbž'UbU'mg]g'UbX ]bhYfdfYhUhjcb'cZfYgi 'hg''H\YfY'UfY'a i 'hjd'Y'dcgg]V'Y'gc'i hjcbg'cf'WcbWi g]cbg'Uhh\Y'ci hgYhUbX'gi ZZJW]Ybh Wca d'Yl ]mhc'fYei ]fY'U'XY[ fYY'cZdfc YWhd'Ubb]b[ "'H\Y'h\Yg]g'fYei ]fYg'mci 'hc'Zcfa i 'UhY'dfcV'Ya g']b gW]Ybh]ZJWcf'Yb[]bYYf]b[ 'hYfa gž'a UbU[ Y'Ub'hYW\b]WU''dfc YWhUbX'Z]bX'gc'i hjcbg'VmUdd'm]b[ 'gW]Ybh]ZJWUbX Yb[]bYYf]b[ 'a Yh\cXg''Mci 'k]`''U'gc'XYj Y'cd'mci f'UV]`]mhc'k cf\_']b'U'fYgYUfW\'UbX'XYj Y'cda Ybh Ybj ]fcba Ybh''Mci 'a i gh]XYbh]ZmU'gi dYfj ]gcf'UbX'dfc'YWhdf]cf'hc Ybfc'`]b[ ']b'h\]g'Wci fgY"

#### 7 ci fgY 5]a g

H\Y'h\Yg]g'dfcj ]XYg'Ub'cddcfhi b]hmZcf'h\Y'ghi XYbhhc'Vf]b[ 'hc[Yh\Yf'Yb[]bYYf]b[ 'df]bWjd'Yg'YUfbYX'cj Yf h\Y]f'dfYj ]ci g'mYUfg'cZghi XmUbX'Udd`mh\YgY'df]bWjd`Yg'hc']bbcj Uhji Y`mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y`cda YbhcZU'gdYWjZJWXYg][ bž'dfcW\'gg'UbX#cf'h\Y']bj Ygh][ Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc'YWg'a i gh VY'Wca d'YI ž'cdYb!YbXYX'dfcV'Ya g'l\UhU\'ck 'fcca 'Zcf'ghi XYbhWYUhji ]hmž'UbX'h\Y'UWei ]g]hjcbž'UbU'mg]g UbX']bhYfdfYhUhjcb'cZfYgi 'hg''H\YfY'a i ghVY'a i 'hjd'Y'dcgg]V'Y'gc'i hjcbg'cf'WcbWi g]cbg'Uhh\Y'ci hgYhUbX gi ZJW]YbhWca d'YI ]hmhc'fYei ]fY'U'XY[ fYY'cZdfc'YWid'Ubb]b[ 'Zfca 'h\Y'ghi XYbh''H\Y'h\Yg]g'fYei ]fYg'h\Y ghi XYbhhc 'Zcfa i 'UhY'dfcV'Ya g']b'Yb[]bYYf]b[ 'hYfa gž'a UbU[Y'Ub'Yb[]bYYf]b[ 'dfc'YWiUbX'Z]bX'gc'i hjcbg'Vm Udd`mjb[ 'Yb[]bYYf]b[ 'a Yh\cXg''Ghi XYbhg'U`gc'XYj Y`cd'h\Y]f'UV]`]hmhc'k cf\_']b'U'fYgYUfW\'UbX'XYj Y`cda Ybh Ybj ]fcba Ybh'

## 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" 8 Yj Y`cd`U`XYg][ b`cf`U`dfcWfgg`cf`]bj Ygh][ UhY`U`\mdch\Yg]g`Zc``ck ]b[`]bXi ghfmUbX`dfcZ/gg]cbU` Yb[ ]bYYf]b[`qhUbXUfXq"`
- &" 7 f]h]wU``mfYZYWicb'U'gdYW]U`]ghiVcXmcZ\_bck `YX[Y'fY`UhYX'hc'h\Y]f'h\Yg]g'hcd]w''
- ' " 5dd`mgWYbhjZWUbX'Yb[]bYYf]b[ a Yh\cXg'hc'gc`j Y'Ub'Yb[]bYYf]b[ 'dfcV`Ya "'
- ("5bU`mgY`XUhU`cV^YWhji Y`mi g]b[ei UbhjhUhji Y`UbX`a Uh\Ya UhjWU``a Yh\cXg"`
- ) "8 Ya cbglfUlY cfU`UbX k f]lhYb Wca a i b]WUljcb ]b dfcZ/gg]cbU`UbX `UmXca U]bg"
- \*" Gc`j Y`V]ca YX]WU``dfcV`Ya g`Vm'Udd`m]b[ '7 @C g'%l) "

### HYUWX]b[ GhfUhY[]Yg

H\Y`Wci fgY`]g`HJi [\h'Ug`Ub`]bX]j ]Xi U``fYgYUfW\`dfc`YWfz`hc`XYj Y`cd`U``Yj Y``cZfYgYUfW\`g\_]``g`UbX Ui hcbca m'

## 5XX]h]cbU~7ci fgY ±bZcfa Uh]cb

H\YfY`]gʻbcʻcZZMJU``WUggʻha YʻZcfʻh\]gʻWci fgYžk\]W\`d`UWYgʻUʻZcW gʻcbʻgY`Za chji UhYX``YUfb]b["'Mci 'a i ghgh]``Ybgi fYʻmci fʻYbfc`a YbhʻUbXʻfY[]ghfUhjcbʻ]gʻi dʻhcʻXUhYʻ]bʻmci fʻYbfc`a YbhʻMci 'a i gh\Uj YʻgY`YWYX`U dfc YWhVYZcfY`K YY\_\_\$'cZhYfa "'=Zmci `\Uj YbfhiXcbY`gc`UʻfYUXmž'd`YUgY`WcbhUWhh\Y`Wci fgY`WccfX]bUhcf" Mci fʻZJWY!hc!ZJWYʻhja YʻbYYXgʻhcʻVYʻcf[Ub]gYX`k]h\`mci fʻgi dYfj ]gcfž'Ugʻmci 'UfY`YI dYWYX`hcʻa YYhh\Ya 'Uh`YUghcbWYʻdYf`k YY\_"'

#### 91 dYWHJh]cbg cZGhi XYbhg

- EWX\_af[hSfW Vsd [`Y
- ? WWf kagdegbWth[eaddWYg/Sd/k
- 5a\_bWfWS^fZWSeeWee\_W fe a`f[\_W
- 3 ffW V fZWI WW] \$ fgfad[S^a` Ua`VgUf[`Y '[fWdSfgdWdWh[W e

## 5ggYgga Ybh

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	7 ci fgY @YUfb]b[ Ci hWca Yg 5ggYggYX
%" ±bhYf]a 'F Ydcfh	%\$ı	A cbXUmK YY_'‱'‱) - 'da	%ž'&ž' ž'( ž') ž'*

#### 5ggYgga Ybh% ⇒bhYf]a FYdcfh

Gi Va ]gg]cb bch/g. Mci a i ghgi Va ]hiU XfUZhcZmci f@]hYfUhi fY F Yj ]Yk hc Ga Ufh\]b\_]b[ žh\Yb gi Va ]hmci f Ga Ufh\]b\_]b[ ZYYXVUW\_hc A ccX`Yždf]cf hc gi Va ]gg]cb cZmci f \( \delta\)hYf]a F Ydcfh' C bW mci fYWY]j Y h\Y ga Ufh\]b\_]b[ ZYYXVUW\_žd`YUgY i d`cUX h\]g hc A ccX`YžUbX h\Y gi Va ]gg]cb hcc`k ]``cdYb Zcfmci " 8i Y`XUhY. A cbXUmK YY\_'\%\%\) - da

 $\pm$   $\hbar Yg]gž \hbar YfY UfY <math>\hbar fYY X]ZYfYbh UggYgga YbhghU[Yg"]$ 

H\Yq]q·5. ±bHYf]a FYdcfhif%\$ı ŁžGi dYfj ]qcf·7\YW\_`]qhifG5# BŁ

H\Yg]g'6. 'Dfc[fYgg'GYa]bUf'UbX'FYZYWjcb'f%\$i Ł

H (Yg) g (7.1) b U (Ydcfhift) + L (DUfh) W (Uh) c b (f) + L (2.7 c b Z (f Y b W (D f Y g Y b H Uh) c b (f W (b L L Uh) c b (f W (b L Uh) c b

#### Mci f'cVYWnjj Yg']b H\Yg]g'5 UfY

%" Hc`Vi]`X`mcif`\_bck`YX[Y`VUgY`cb`mcif`gdYW]Z]WfYgYUfW\`hcd]W' &" IgY`h\Uh\_bck`YX[Y`VUgY`hc`]bZcfa`mcif`gdYV]Z]WdfcYWhU]ag`UbX`aYh\cXc`c[m' '" 7 cad`YhY`dfY`]a]bUfmk cf\_`hck UfXg`aYYh]b[`h\Y`gdYW]Z]WdfcYWhU]ag"

Hc 'UggYgg'h\]gž'h\YfY'UfY'hk c 'UggYgga Ybh'hUg\_g'Zcf'H\Yg]g'5"P

#### ±bhYf]a 'fYdcfh'f%\$ı Ł

# %" >i ghļ<br/>ZļWuhļcb`cZhja Y`U``cWuhļcb`Zcf`YUVX`hUg\_ ' " 5j U]`UV`Y`fYgci f<br/>V\Wg`]XYbhjZļYX

- (\*FYei]fYX'ffU]b]b['UbX'idg\_]``]b[']XYbf]Z]YX
- % Dfc YWn8 YdYbXYbhDfYdUfUhcbg f&\$1 Ł
  - %" 7 Ub 'h\Y 'gh XYbh'UW\]Yj Y 'h\Y 'U]a g ']b 'h\Y 'h]a Y`]bY3 'K \Uhdfc[ fYgg '\Ug 'VYYb 'a UXY U`fYUXm3
  - &" Dfc YWngdYWZWEVI ha Um]bWi XY
    - %" 9j ]XYbW cZlfU]b]b[ cb gdYW]ZWYei ]da Ybh
    - &" 9j ]XYbWY cZgca Y'i dg\_]``]b[ ']b'bYk 'gcZk UfY#a Yh\cXg
    - ' " DfY`]a ]bUfmifYgi `hg
    - ("DfY`]a]bUfmig\_YhWXYg
    - ) " 7 ca dcbYbhg#dUfhg cfXYfYX
    - \*"8YHJ]YX'ViX[Yh" **†**

‰

‰

## F Ygci fWYg

#### DfYgWf]VYX F Ygci fWYg

F Ygci fWg'k]``VY'a UXY'Uj U]`UV`Y'hc'\Y`d'ghi XYbhg'[i]XY'h\Ya']b'h\Y]f''ci fbYmZcf'H\Yg]g'5"

# 91 hybg]cbg

 $\label{thm:condition} $$Mci `WUb`Udd`m'Zcf'gdYW_U``Wcbg]XYfUh]cb'k \Yb']``bYgg'cf'ch\Yf'W]fW a glUbW'g']bhYfZYfY'k ]h\'mci f UggYgga Ybh'dYfZcfa UbW'''$ 

Ch\Yf'Udd`]\Wh]cbg'Zcf'YI hYbg]cb'cZgi Va ]gg]cb'cZh\Yg]g'fYdcfhg'fY"[ "'Yei ]da Ybh'VfYU\_Xck bžYh\WL

%" 8 ]gW gg h\Y dcgg]V]`]micZUb YI hYbg]cb k ]h\ inci f gi dYfj ]gcf Z[fgh"

- &" F Yei Yghg`WUb`h\Yb`VY``cX[YX`Vmh\Y`gh XYbh\\YfY`<u>\hd.##]bmi f`"Wca #mm&^ndmj</u>."'H\Y`gi dYfj ]gcf`k ]`` h\Yb`fYWY]j Y`Ub`Ya U]``Ug\_]b[`h\Ya`hc`Uddfcj YžVYZcfY`]h`]g`YgWU`UhYX`hc`h\Y`XYV][g]cb`dUbY`"
- "FYeiYghaighVY`cX[YXVmKYY\_\*cZhYfa"
- ("DUbY`XYW]g]cb'k]``VY'a UXY'VmYbX'cZkYY\_'+"
- ) " H\Y`XYV][g]cb`k]``VY`a UXY`VmU`dUbY``—`Wcbg]gh]b[ `cZh\Y`<cG`fcf`h\Y]f`bca ]bYYŁž'H\Yg]g 7 ccfX]bUhcfž'UbX`%ch\Yf`dYfgcb"
- \*" Ghi XYbhgʻg\ci `X`VY`U`YfhYX`hc`h\Y`ZJ\Wih\Uhih\]gʻ]gʻbch[i UfUbhYYXž'UbX`h\i gʻg\ci `X`bchfY`mcb [Yhh]b['Ub`YI hYbg]cb"
- +" Hmd]\W\``m\`Y\ hYbg]cbg`UfY`[fUbhYX`ID`HC`'`kYY\_g"`H\Y``Yb[h\`cZh\Y`Y\hYbg]cb`bYYXg`hc`VYfYeiYghYX`UbX`^gh]\Z]YX`Vmh\Y`gidYfj]gcf"`DUbY``k]```XYW]XY`h\Y``Yb[h\`cZh]aY`[fUbhYX"

# DfcWYXi fY ]Zmci ZU] H\Yg]g 5 ž 6 cf 7

: U]``]b`H\Yg]g`6`fgYa ]bUf`a Uf\_O`)\$ı Ł-aighfY!Ybfc``]b`H\Yg]g`6`U[U]b

 $: U]^{\ }]b H Yg]g 7 - Gh XYbhg \Uj Y h fYY cdh]cbg"$ 

%" fY!Ybfc`Zcf'H\Yg]g'5ž6'UbX'7'U[U]bžbYk'dfcYWhUbX'gi dYfj ]gcf

&"fy!Ybfc``Zcf'H\Yg]g'7'U[U]bžgUaY'dfcYWi!'bYYXg'WcbgYbhcZUb'Uddfcdf]UhY'gi dYfj]gcf/'gh XYbh'' Gh XYbhXcYg'Z fh\Yf'k cf\_žfY!gi Va]hg'h\Yg]g'UZhYf'U'aUl'cZ\*'k YY\_g"'5agæWaUf\_'WUddYX'Uh)\$\"'-Zgh]```bchgUh]gZUWcfmž'h\Yb'bYYXg'hc'fY!Ybfc`"

 $H\ [g``Ughcdh]cb`]g`cb`mUj\ U]`UV`Y`]Zh\ Y`cf][\ ]bU``a\ Uf\_'k\ Ug`\ (\ \$\check{z}'CF\ `]Zh\ Y`ghi\ XYbh']g`]b`h\ Y]f``UghgYa\ YghYf\ VYZcfY`[\ fUXi\ Uh]cb`ffY[\ UfX`Ygg`cZh\ Y`cf][\ ]bU``a\ Uf\_Ł"$ 

: U]``]b`H\Yg]g`6'/ '7'fk\Yb'fU\_Yb`g]a i `HJbYci g`mŁ-'Gh XYbhg`a i ghfY!Ybfc``]b`H\Yg]g`6'U[U]bž'UbX WUbbchWcbW ffYbhmYbfc``]b'7"'H\YmWUb'h\Yb'fU\_Y'H\Yg]g'7'k\Yb'H\Yg]g'6'\Ug'VYYb'gUhjgZJWcf]`m Wca d`YhYX"

# ±bXi ghfmH\YgYg

K Y YbWci fu[ Y gh XYbhg hc gYY\_dufhbYfg\]dg k ]h\ ]bXi ghfmžgc gh XYbhg WUb \Uj Y U Wc!gi dYfj ]gcf Zfca

]bXi glfm" < ck Yj Yfž ]ZWcbZ]XYbh]U`]hm]g fYei	i]fYXžU`WcbZ]XYbh]U``X]gWcg	gi fY`U[fYYa Ybhff785½]g`cV`][Uhcfm

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

@UVcfUlrcfmfYdcflrg\*UbX\*a U\*cf\*Ugg][ ba Yblrg\*k ]\*\*\*fYei ]fY\*U\*Bcb\*D\*U[ ]Uf]ga \*8YWUfUlr]cb\*7cj Yf\*G\YYlr"

5gg][ ba Ybhgʻg\ci `X`VYʻgi Va ]hhYXʻcbʻhja Y"`5`XU]`mdYbU`hmcZ) ı `cZh\Y`a Uf\_gʻUj U]`UV`YʻZcfʻh\Uh Ugg][ ba Ybhk ]```Udd`m'Zcfʻk cf\_ fYVW]j YX`UZhYfʻh\Y`Xi Y`XUhY"`5bmUgg][ ba Ybha cfYʻh\Ubʻ) `XUmg``UhYʻk ]``bch VY`UWWdhYX"'H\Y`cb`mYl Ya dhjcbʻk ]```VYʻk \Ybʻdf]cfʻdYfa ]gg]cbʻZcf``UhYʻgi Va ]gg]cbʻ\UgʻVYYbʻ[ fUbhYX`Vm h\Y`7 ci fgY`WccfX]bUhcf"'9I hYbg]cbgʻk ]```VY`[ fUbhYX`cb`mcbʻa YX]WU``cf`Wca dUgg]cbUhYʻ[ fci bXgʻi bXYf YI hYa Y`W]fW a ghUbWfg"

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D@5; =5F =€A

6Yk UfY"5b'Ugg][ ba Ybhih Uh]bWi XYg'd'U[ ]Uf]gYX'a UhYf]U`'k ]``'fYWY]j Y'U'\$i ': U]`ž'UbX'ghi XYbhg'k \c d'U[ ]Uf]gY'k ]``'\Uj Y'h\Y]f'bUa Yg'YbhYfYX'cb'U'd'U[ ]Uf]ga fY[ ]ghYf'UbX'k ]``'\Uj Y'h\Y]f'bUa Yg'YbhYfYX'cb'U'd'U[ ]Uf]ga fY[ ]ghYf'UbX'k ]``'VY'`]UV'Y'hc'X]gW]d']bUfmUW]cbž']bWi X]b[ 'YI Wi g]cb'Zfca 'Ybfc`a Ybh'''

♣i]g'YI dYWYX'h\Uh'U``'ghi XYbhg'a i ghUh'U``'h]a Yg'gi Va ]hh\Y]f'ck b'k cf\_'Zcf'UggYgga Ybh''Gi Va ]hh\b (h\Y k cf\_'cf']XYUg'cZgca YcbY'Y`gY'k ]h\ci h\WYUf'mUW\_bck `YX[ ]b[ 'h\Y'gci fW'cZVcffck YX'a UhYf]U``cf']XYUg']g d`U[ ]Uf]ga "

## 5WUXYa]W±bZcfa Uh]cb

#### 7CI FG9'9J5@ 5H€B'5B8'89J9@CDA 9BH

Gh XYbhZYXVUW\_`\Ug`\Y`dYX'hc`g\UdY`UbX'XYj Y`cd'h\]g`Wci fgYž]bWi X]b[ 'ZYYXVUW\_`cVhU]bYX'Zfca 'cb! `]bY'Yj U`i Uh]cbg'Ug'dUfhcZl BGK 'g'Ug'dUfhcZl BGK 'g'a m9l dYf]YbW'dfcW\gg\"'Mci 'UfY`\][\`m YbWci fU[ YX'hc'Wca d`YhY'gi W\'Ub'cb!`]bY'Yj U`i Uh]cb'hck UfX'h\Y'YbX'cZHYfa "': YYXVUW\_'UbX'gi [ [ Ygh]cbg dfcj ]XYX'k ]``VY']a dcfhUbh]b']a dfcj ]b[ 'h\Y'Wci fgY'Zcf'Z hi fY'gh XYbhg"

85 H9G HC BCH9
FYZYf hc A m BGK Zcf = dcfhUbh8UhYgž Uj Uj UV Y Uh