

"k Ufa]b[ž'UbX nk]h\ "ck "Yj Y`g'cZ c`hJ]W X'gmghYa g" Mci 'k]` [YbYfUh]cb fUhcfmYl YfW]gYg h]cb'cZgc`Uf cc``UbX'i gY']hhc a g" @YWi fY gYgg]cbg k]```]blfcXi WY h\Ycfm'UbX'k cf_YX'Yl Ua d`Yg"'K cf_g\cdg'UfY`XYg][bYX'tc`XYj Y`cd fY`Yj Ubh'dfcV`Ya 'gc'j]b['g_]``g"8i f]b['gca Y'k cf_g\cd'WUggYgžghi XYbhg'k]```[c'h\fci [\ ``UV'Yl YfW]gYg UbX'UggcW]Uh'X`Ugg][ba Ybhg'k \]W\ k]```\Y`d'ghi XYbhg'tc`XYj Y`cd'g_]``g'fY`Uh'X`tc'h\Yi gY`cZgcZk UfY`Zcf a cXY`]b['gc`Uf'WY``g'UbX'a cXi `Ygž'dfUWj]WU``g_]``g'fY`Uh'XX'tc'UggYa V`]b['UbX'a YUgi f]b['th\Y'dYfZcfa UbW' cZd\chcj c`hU]WgmghYa g'UbX'g_]``g'fY`Uh'XX'tc']bhYfdfYh]b['Yl dYf]a YbhU``fYgi 'hg"'E i]nnYg'UfY`XYg][bYX'tc \Y`d'k]h\ Wcbh]bi ci g``YUfb]b['UbX'_bck `YX[Y`Yb\UbWa Ybh''`H\Y'DJ 'gmghYa 'XYg][b'Ugg][ba Ybhik]``\Y`d hc`XYj Y`cd'g_]``g'bYYXYX'Zcf'DJ 'gmghYa g'XYg][b"

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Ghi XYbhgʻa i ghiXYa cbghfuhYʻh\YmWubʻXYg][bʻuʻghubX!UʻcbYʻDJʻgmghYaʻZcfʻcdh]a i a'dYfZcfa UbWYʻubX WcbZcfa UMXXXZthTaffUbWYʻhcʻfY`Yj Ubhʻ5 i ghfuʻ]UbʻGhubXUfXg"ʻ5ʻa]b]a i a'a Uf_ʻcZ*\$i ai ghiVYʻcVhJ]bYXʻ]bʻh\YʻDJ gmghYaʻXYg][bʻUgg][ba Ybhʻ]bʻcfXYfʻhcʻdUggʻh\]gʻWci fgY"ʻ: U]`i fYʻhcʻUW\]Yj Yʻh\]gʻa]b]a i a'a Uf_ʻk]``fYgi`h]bʻUbʻi bgUhjgZUWhcfmZJ]`fI: Ł'[fUXYžfY[UfX`YggʻcZh\YʻdYfZcfa UbWY`]bʻh\YʻfYghcZh\YʻWci fgY"

5 hmy b X Ub WY F Yei] f Ya Yb hg

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A ccXY. '5g'U'dUfficZf\Y'fYUW\]b['Wca dcbYbhžA ccXY'k]``VY'i gYX'fc'X]ggYa]bUfY'fYUW\]b['a UfYf]U'gž\cghZcfi a g'UbX'cWWg]cbU``mei]nnYg"'5bbci bWYaYbhg'WcbWYfb]b['Wci fgY']bZcfa UfJcb'k]``VY'[]j Yb']bf\Y'`YWhfYg'UbX#cf'cb'AccXY"'5ggYggaYbhaUf_g'k]``'U`gc'VY'aUXY'UjU]`UV'Y'j]U'AccXY.\fhdg.##accXY"fY'hY'h'i bgk "YXi"Ui #c[]b#bXYI"d\d"

7ci fgY'9j U'i Uh]cb'UbX'8Yj Y'cda Ybh

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±0 'fh`]g'Wci fgYžfYWYbh']a dfcj Ya Ybhg'fYgi `h]b['Z̄ca 'ghi XYbh'ZYYXVUW_']bWi XY'fYa cj U`cZbcb!Wf]h]WU` ei Ygh]cbg'Z̄ca '`UV'Ugg][ba Ybhgž'UbX']bWfYUg]b['fh`Y'k Y][\h]b['Z̄cf'fh`Y'DJ 'ghUbX!U`cbY'DJ 'gmghYa 'XYg][b Ugg][ba Ybh'':]bU`YI Ua '\Ug'U`gc'VYYb'fYa cj YX"

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK `ghiXYbhg`UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``Ha Y'a UbU[Ya Ybh']g'Ub']a dcffUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbhJZYX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'ha Y'a UbU[Ya Ybh''Gh XYbhg'g\ci `X'U``ck 'gi ZZWYbh'Ha Y'Zcf fYgYUfW\žXfUZf]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fWYg']b'dfYdUf]b['U``UggYgga Ybh'HJg_g"

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

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5ddYbX]I . '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

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