

The University of New South Wales
 School of Mathematics and Statistics
 Mathematics Drop-in Centre

ALGEBRAIC IDENTITIES

There are a number of algebraic identities which you need to know in order to help you solve equations and simplify expressions (by an *identity* we mean an equation involving one or more variables, which is true for all values of those variables).

- Addition of fractions:

$$\frac{w}{x} + \frac{y}{z} = \frac{wz + xy}{xz} .$$

- Square of a sum:

$$(x + y)^2 = x^2 + 2xy + y^2 .$$

- The same for a sum of three terms:

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz .$$

- Difference of two squares:

$$x^2 - y^2 = (x - y)(x + y) .$$

- Difference of two powers:

$$x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \dots + xy^{n-2} + y^{n-1}) .$$

$$y^n - x^n = (y - x)(y^{n-1} + y^{n-2}x + \dots + xy^{n-2} + x^{n-1}) .$$

Note that there is *no* formula for factoring the sum of two squares, for example, $x^2 + y^2$ cannot be factored.

- The Binomial Theorem to be dealt with in a separate sheet.
- Power laws and logarithms are dealt with in separate sheets.

Comments.

- You must be able to use the identities to factorise expressions. For example, if you see $x^2 - 5$, you should be able to factorise it as $(x - \sqrt{5})(x + \sqrt{5})$, and if you see $x^2 - 5$, you should be able to factorise it as $(x - \sqrt{5})(x + \sqrt{5})$.
- You must be able to replace variables by different variables, for example, $x^2 - 5$ can be written as $m^2 - n^2$, where $m = x$ and $n = \sqrt{5}$.

$$m^2 - n^2 = (m - n)(m + n)$$

$$x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5})$$

$$9a^2 - 100b^2 = (3a - 10b)(3a + 10b)$$

$$x^6 - y^6 = (x^3 - y^3)(x^3 + y^3) .$$

- Sometimes we can use a number of these identities successively in order to give a detailed factorisation of a certain expression. For instance,

$$x^6 - y^6 = (x^3 - y^3)(x^3 + y^3)$$

EXERCISES.

Please try to complete the following exercises. Remember that you cannot expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start,