The University of New South Wales
School of Mathematics and Statistics
Mathematics Drop–in Centre

LOGARITHMS

"Logarithm" is really just another word for "exponent" or "power". In an expression a^b we call a the base and b the logarithm. So if

$$a^b = c$$

then we say that b is the logarithm of c to the base a, written

$$b = \log_a c \, .$$

In all of these equations, a and c must be positive numbers; b may be positive, negative or zero. In principle, logarithms can be evaluated by rewriting them as powers.

Example. Evaluate log 32.

Solution. Let x = log 32. Then $2^x = 32$ and so by trial and error $x = 5$.

In practice, evaluating logarithms usually requires a calculator. For instance, if x = log 34 then $2^x = 34$; the previous example shows that x must be a bit more than 5 and definitely less than 6, but it's difficult to pin it down much more closely than this.

Logarithm laws. The following identities hold, where a, x, y are positive numbers and p is any real number:

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad (□)$$

log

EXERCISES.

Please try to complete the following exercises. Remember that you cannot expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Write the following statements as equations involving logarithms:

   (a) $1331 = 11^3$; (b) $25^{1/\ } = 5$; (c) $y = x^{\pi}$;

   (d) $p^q = 7$; (e) $2^x = \frac{3}{\ }$; (f) $z^{-1\cdot\ 3} = 4{\cdot}56$.

2. Write the following as equations involving powers:

   (a) $7 = \log_3 2187$; (b) $x = \log_1\ \ 2$; (c) $\log\ \ a = -3$;

   (d) $x = \ln 5$; (e) $\ln x = 5$.

3. Evaluate, without using a calculator,

   (a) $\log\ \ 128$; (b) $\log_1\ \ \ 5$; (c) $\log_1\ \ \frac{1}{1}$.

4. Simplify if possible

   (a) $\log\left(\frac{x}{y}\right) + \log\left(\frac{8y}{x^9}\right)$; (b) $3^{\log_3 x - \ \log_3 y}$;

   (c) $\log(x\ \ + y\ \ )$; (d) $4\ln\left(\frac{s}{t}\right) + \ln\left(\frac{s}{t}\right)$;

   (e) $\log_1\ (2^z 3\ ^z)$; (f) $(\ln a)(\ln b)$;

   (g) $\ln 10 + \ln 100 + \ln 1000$; (h) $\ln\left((xy)^{-3}\right) - \ln\left(\frac{x}{y}\right)$.

ANSWERS.

1. (a) $\log_{11} 1331 = 3$;

   (b) $\log\ \ \ 5 = \frac{1}{\ }$;

   (c) $\log_x y = \pi$;

   (d) $\log_p 7 = q$;

   (e) $x = \log\ \ \frac{3}{\ }$;

   (f) $\log_z 4{\cdot}56 = -1{\cdot}23$.

2. (a) $3\ \ \ = 2187$;

   (b) $10^x = 2$;

   (c) $a = 5^{-3}$;

   (d) $e^x = 5$;

   (e) $x = e\ $.

3. (a) 7; (b) $\frac{1}{3}$; (c) -2.

4. (a) $3 - 7\log\ x + 2\log\ y$, or $3 + \log\left(\frac{y}{x}\right)$;

   (b) $\frac{x}{y}$;

   (c) no simplification;

   (d) $27\ln s - 32\ln t$, or $\ln\left(\frac{s}{t^3}\right)$;

   (e) z;

   (f) no simplification; the given expression can be written as $\ln(b^{\ln a})$ or $\ln(a^{\ln b})$, but these are not really simpler;

   (g) $6\ln 10$;

   (h) $-9\ln x + 4\ln y$, or $\ln\left(\frac{y^4}{x^9}\right)$.